# Comparison of Microwave and Mutual Inductance Measurements of the Inductive Nonlinearity of HTS Thin Films

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Abstract—A quadratic dependence of penetration depth on supercurrent density may be observed both by an inductive measurement on unpatterned films or by a microwave measurement of third harmonic generation in coplanar waveguide. Results for the two methods are compared for a series of YBCO films. We find good agreement between the two measurements.

#### I. INTRODUCTION

One important contribution to nonlinearity in superconducting circuits is the current dependence of the penetration depth  $\lambda$ . This is predicted by BCS theory to have the form

$$\lambda^2(T,J) = \lambda^2(T,0)[1 + (J/J_0)^2],$$
 (1)

where J is the current density. The parameter  $J_o$  sets the current scale for which nonlinear effects become important. Calculations of  $J_o$  as a function of temperature have been presented for the case of a BCS superconductor [1].

Fits to third harmonic generation in coplanar waveguide made with YBCO films show that (1) can account for the data over a range of guide dimensions and length [2]. On the other hand, measurements of intermodulation distortion in microstrip resonators made from TBCCO films [3] could be better fit to a dependence

$$\lambda^{2}(T,J) = \lambda^{2}(T,0)[1+|J/J_{0}|].$$
 (2)

In the microwave measurements rf currents are concentrated near patterned edges of the HTS films, where secondary effects such as flux entry, degraded superconducting properties, etc, could be important. It would be valuable to be able to independently measure the  $\lambda(J)$  dependence of films prior to their being patterned and incorporated in a microwave circuit. We show here that a well-known mutual inductance technique used to measure the penetration depth in unpatterned films [4] can be extended to obtain a measure of the current dependence of  $\lambda$ . Preliminary comparisons of companion films to those used in third harmonic generation studies show good agreement between the inferred  $J_{\rm o}$  values.

# II. MEASUREMENT METHOD

To make our inductive measurement, we combine a small ac current (typically 10 kHz) with a dc current component in a drive coil. The ac voltage across a secondary coil is detected with a lock-in amplifier. The two coils are positioned on opposite sides of the film on a common axis. It is shown in [4] that the best results are obtained if the coils are small compared to the film extent and are as thin as possible in the axial direction--that is, have the shape of a washer.

The ac coupling between the coils is purely inductive except very close to the transition temperature. With knowledge of the coil parameters and film thickness, the measured mutual inductance may be used to infer the penetration depth of the superconductor [4].

The dc current in the drive coil induces a screening supercurrent in the superconducting film. The radial dependence J(r) of the screening current in the film may be calculated with good accuracy by assuming that it completely screens magnetic fields from the reverse side [5]. (Actually some field "leaks" through the film and indeed this is the basis for the measurement of  $\lambda$ . However, in practical cases this represents a small correction to J(r).) Fig. 1 shows a calculation of J(r) for the coil used in our experiment, assuming the coil is pressed against the film side of the sample. It can be seen that J is peaked with a maximum near the mean radius of the coil windings and dropping off fairly rapidly with increasing r.

If  $\lambda$  depends on J, application of a dc current to the drive coil results in a sample whose penetration depth is not homogeneous. If the variations in  $\lambda$  across the film are small, it can be shown that the mutual inductance measurement yields a weighted average of  $\lambda(r)$ . It is plausible that the strongest weighting is just where the screening current has its maximum. In what follows, we assume that the measured penetration depth corresponds to the <u>maximum</u> screening current J. There is of course some error in this approximation, and we do not claim that one has a precision  $\lambda(J)$  measurement by this technique. However it surely will yield numerical values of adequate accuracy for purposes of selection and comparison, as well as establishing the form of the  $\lambda(J)$  relationship.

# III. HEATING EFFECTS

In order to induce film currents large enough to significantly effect  $\lambda$ , a rather large dc coil current is frequently needed. The power dissipated in the coil causes the film to heat up, and since  $\lambda$  depends on temperature this gives

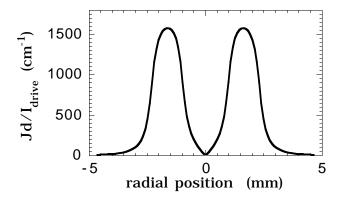


Fig. 1 The calculated radial dependence of the screening sheet current Jd (d is the film thickness) in the superconducting film normalized to the dc drive current in the coil used in our experiment.

a spurious effect unrelated to the  $\lambda(J)$  dependence of interest. Heating effects turn out to be significant only in the thickest HTS films of interest. (The coil current required to generate a given current density in the film is proportional to the film thickness, cf. Fig. 1). We can nevertheless make measurements with our existing mutual inductance setup provided two conditions are met: (a)  $\lambda(J)$  must have the quadratic form of (1), and (b) the film must have strong pinning. We first cool the film through its transition with a current  $I_{trap}$  in the drive coil. It can be shown [6] that if pinning is strong, the current density in the film is subsequently proportional to  $(I_{coil} - I_{trap})$ . We then fit our data to

$$\Delta \lambda = AI_{coil}^2 + B(I_{coil} - I_{trap})^2, \tag{3}$$

where the coefficient A is related to the heating effect and we can obtain  $J_o$  from the coefficient B [6].

A better solution to the heating problem is currently being investigated. A modified drive coil is used, consisting of two interspersed windings. The film current density will then be proportional to the sum of the dc currents in the two windings while the power dissipation is proportional to the sum of their squares. It is possible to keep the latter constant (ensuring a constant film temperature) while varying the former, thus sweeping the film current density at a constant temperature

#### IV. MUTUAL INDUCTANCE RESULTS

Fig. 2 gives representative results for three samples measured by the inductive method. Two of them are YBCO films of thicknesses 50 and 450 nm. These were cooled with an appropriate trapping current, as discussed above. The data shown is the residual after subtracting away the fitted heating term in (3), thus is the intrinsic contribution due to film current J. The heating term for the 450 nm film was  $\sim$ 4 times greater than the effect of current, while in the case of the 50 nm film it was relatively unimportant and could have been ignored with little error. In both cases the dependence on current is quadratic as in (1), with similar values of  $J_0$ .

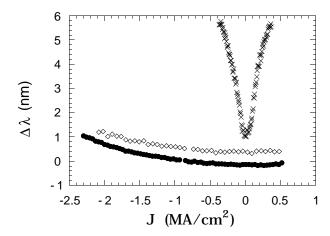


Fig. 2. The change of penetration depth  $\lambda$  as a function of film current density J for various samples referenced in Table 1: solid circles: 11301-2; diamonds, L397-169; crosses: 5264-3. In the first two cases measurements were taken with a trapped film current; the fitted heating contribution to the raw data has been removed. In the last case the estimated heating effect is negligible and there was no trapped current. The data are offset vertically for clarity.

The third sample shown in Fig. 1 was a commercially obtained 650 nm TBCCO film specified as having a low microwave surface resistance. In this case the effect of film current is much larger than that due to heating, and no correction has been made in the data. Clearly the dependence is not quadratic, and perhaps (2) would be a better fit.

Table 1 shows the fitted values for  $J_o$  for all the samples that have been measured inductively to date. Also given are other film parameters at 78 K: the penetration depth and the inductively measured critical current density [5]. Clearly there is a correlation between all these measures of film quality.

# TABLE I

Summary of results on various samples. Thickness and penetration depth are given in nm, and current densities in MA/cm².  $J_c$  values were measured inductively by the technique of Ref. [5]. The rf values of  $J_o$  were obtained from companion samples patterned into coplanar waveguide, and are an average over results from a variety of lengths and waveguide dimensions. The "dc" values were found by the inductive method described here.

| sample # material substrate thickness λ | $J_c (J_o)_{dc} (J_o)_{rf}$ |
|---|-----------------------------|
| L397-169 YBCO LAO 50 360                | 3.4 31 28                   |
| L397-414 YBCO LAO 400 315               | 2.9 35 27                   |
| L397-441 YBCO LAO 400 390               | 2.2 19 18                   |
| 11301-2 YBCO LAO 450 345                | 2.3 28 NA                   |
| 11209-1 YBCO AlO 350 460                | 1.1 5 NA                    |
| 5264-3 TBCCO LAO 650 385                | 0.8 NA <sup>a</sup> NA      |

<sup>&</sup>lt;sup>a</sup>The  $\lambda(J)$  characteristic, shown in Fig. 1, could not be fit to a quadratic dependence.

Some of the samples listed in Table 1 had companions deposited under identical conditions that were made into coplanar waveguides. Assuming the third harmonic generation in these guides is due entirely to a nonlinear penetration depth of the form (1), it is possible to deduce a value of the coefficient  $J_0$  [2]. Table 1 shows that there is rather good correlation between the rf and inductive determinations, for films covering a range of thickness and preparation conditions. Note that the maximum  $J_0$  measured at 78 K,  $\sim 3x10^7$  A/cm², is a factor 2 to 4 smaller than the BCS prediction [1], depending on whether an s-wave or d-wave symmetry of the order parameter is assumed.

# VI. CONCLUSIONS

In the case of the best YBCO films it appears that the  $\lambda(J)$  dependence is quadratic and there is good agreement between microwave and inductive measurements. Since the latter is done on unpatterned films, it may prove useful in vetting films prior to subsequent processing into circuits. For the one TBCCO film we measured, the  $\lambda(J)$  dependence is clearly not quadratic. We have also observed a non-quadratic response in some poorer quality YBCO films. As the observed nonlinearity of even the best films measured is substantially larger than the prediction of the BCS model, there is the possibility of improved film performance with optimized deposition procedures.

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